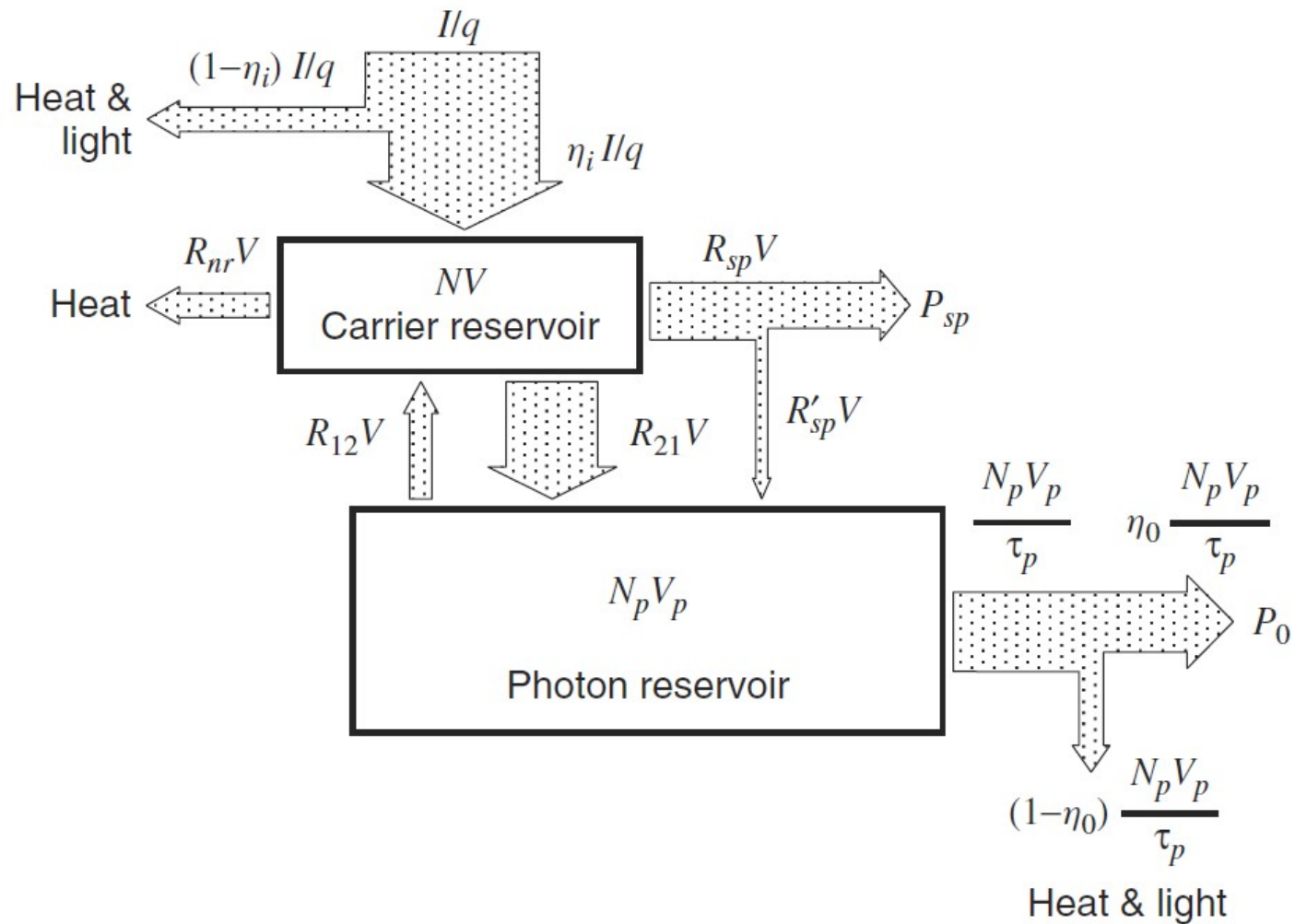


Review of Chapter 2

Number of particles flow

2



The rate equations

3

□ The carrier and photon **number** rate equations

$$V \frac{dN}{dt} = \frac{\eta_i I}{q} - (R_{sp} + R_{nr})V - (R_{21} - R_{12})V$$

$$V_p \frac{dN_p}{dt} = (R_{21} - R_{12})V - \frac{N_p V_p}{\tau_p} + R'_{sp} V$$

□ The carrier and photon **density** rate equations

$$\frac{dN}{dt} = \frac{\eta_i I}{qV} - (R_{sp} + R_{nr}) - v_g g N_p$$

$$\frac{dN_p}{dt} = \left(\Gamma v_g g - \frac{1}{\tau_p} \right) N_p + \Gamma R'_{sp}$$



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The rate equations

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- The output power of the mode

$$P_0 = \eta_0 h\nu \frac{N_p V_p}{\tau_p}$$

$$\eta_0 = F \frac{\alpha_m}{\alpha_m + \alpha_i}; \quad \frac{1}{\tau_p} = \nu_g (\alpha_m + \alpha_i) = \frac{2\pi\nu}{Q}$$

- The total spontaneous emission power from all modes

$$P_{sp} = h\nu R_{sp} V$$

$$R'_{sp} = \beta_{sp} R_{sp}$$

$$\beta_{sp} \approx \beta_{sp_th} = \frac{\Gamma \nu_g g_{th} n_{sp_th}}{\eta_i \eta_r I_{th} / q} = \frac{q n_{sp_th}}{\tau_p \eta_i \eta_r I_{th}}$$



Steady-state equations

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- Treat N as an independent parameter, the solutions of N_p and I is

$$N_p = \frac{\Gamma R'_{sp}}{1/\tau_p - \Gamma v_g g}$$

$$I = \frac{qV}{\eta_i} (R_{sp} + R_{nr} + v_g g N_p)$$

- The threshold gain and threshold carrier density

$$\Gamma v_g g_{th} \equiv \frac{1}{\tau_p} \text{ and } g(N_{th}) = g_{th}$$

- The current expression becomes

$$I = \frac{qV}{\eta_i} (R_{sp} - R'_{sp} + R_{nr} + v_g g_{th} N_p)$$

Well below threshold

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- Well below threshold, using the following condition, the steady-state solutions become

$$N \ll N_{th}, \Gamma v_g g \ll 1 / \tau_p$$

$$N_P \approx \Gamma R'_{sp} \tau_p \approx 0$$

$$I = \frac{qV}{\eta_i} (R_{sp} + R_{nr})$$

and

$$P_0 \approx 0$$

$$P_{sp} = \eta_i \eta_r \frac{h\nu}{q} I$$



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Above threshold

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□ Above threshold, using the following relation, the steady-state solutions become

$$N \approx N_{th}, \Gamma v_g g \approx 1 / \tau_p$$

$$N_p = \frac{R'_{sp_th}}{v_g (g_{th} - g)}$$

$$I = \frac{qV}{\eta_i} (R_{sp_th} + R_{nr_th} + v_g g_{th} N_p)$$

$$I_{th} = \frac{qV}{\eta_i} (R_{sp_th} + R_{nr_th})$$

and

$$P_0 = \eta_i \eta_0 \frac{h\nu}{q} (I - I_{th})$$

$$P_{sp} = \eta_i \eta_r \frac{h\nu}{q} I_{th}$$

□ From the photon density expression, it is shown that N and g never actually reach N_{th} and g_{th} for finite output powers. They remain ever slightly below their threshold values.

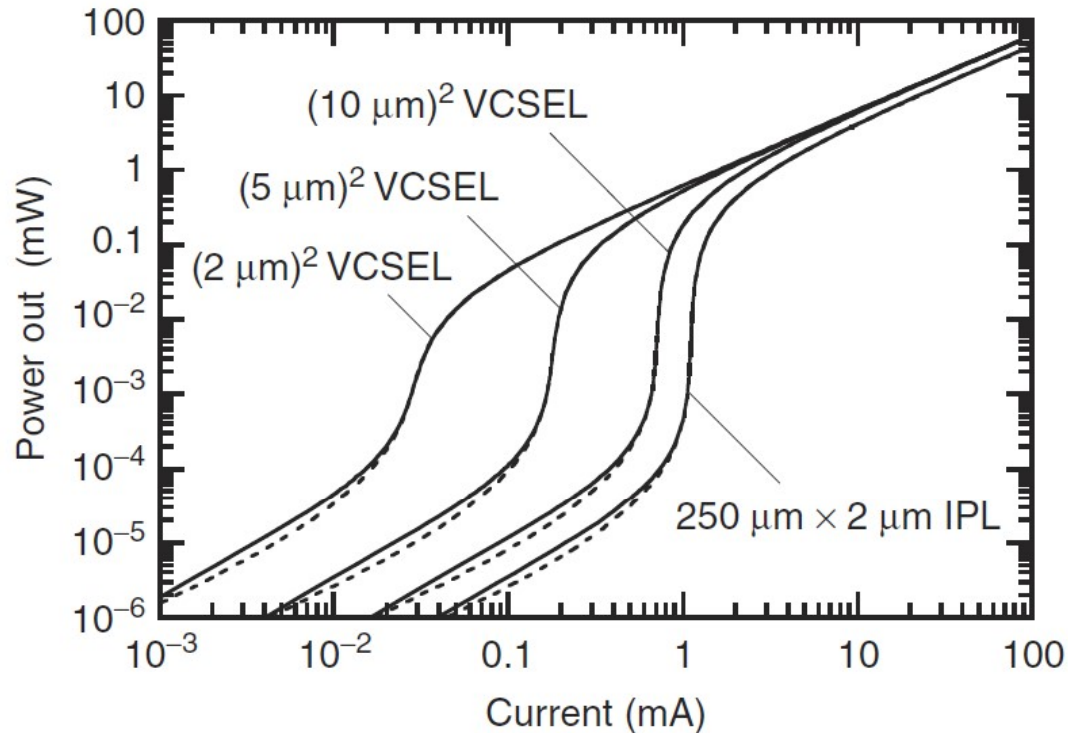
□ When the spontaneous emission factor is large, the more accurate threshold current expression is

$$I_{th} = \frac{qV}{\eta_i} ((1 - \beta_{sp}) R_{sp_th} + R_{nr_th})$$

□ If the non-radiative recom is small, and the spon. factor trends to 1, the laser becomes “thresholdless”.

Below and above threshold

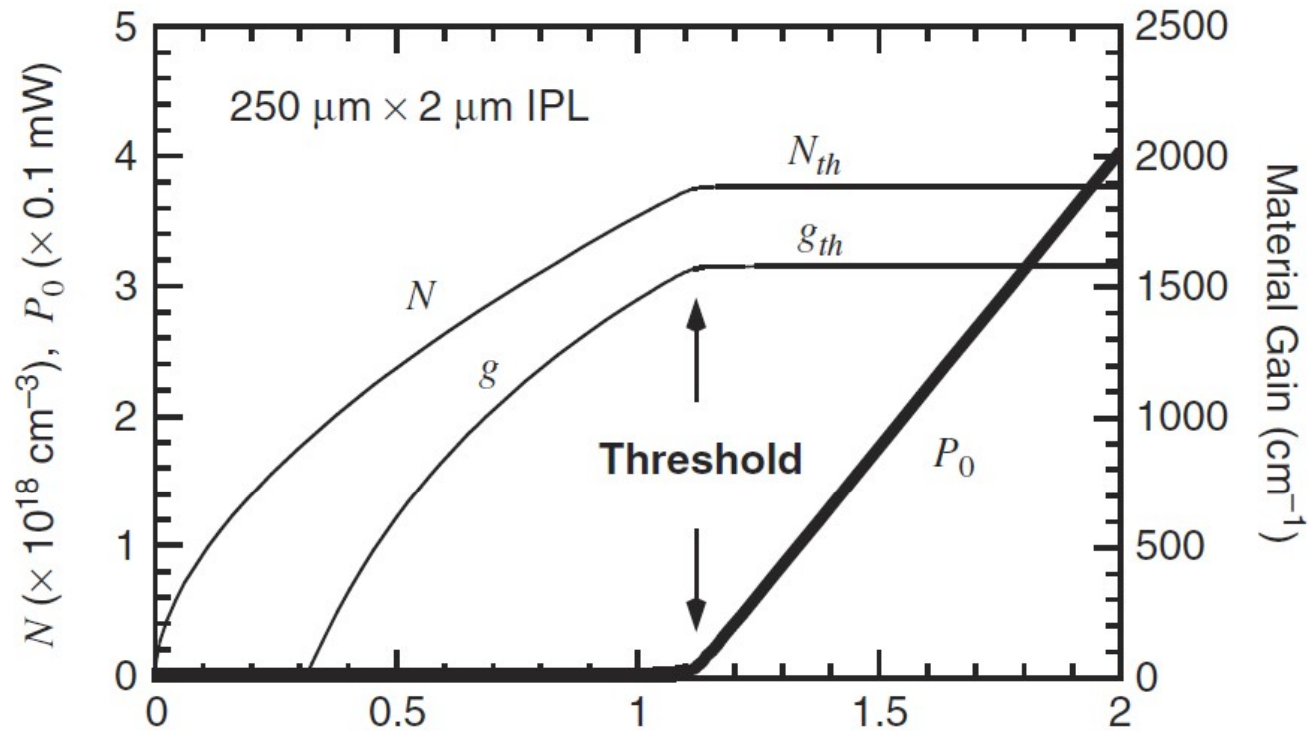
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- ❑ Below the lasing threshold, the spon power is usually less than 1 microW, and is larger for smaller devices, because the spon. emission factor increases.
- ❑ The dashed line indicates that the usage of constant spon. emission factor underestimates the power level at very low current.

Below and above threshold

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- Beyond threshold, the carrier density and gain continue to increase, however, the increase is in the fraction of a percent range. Thus, we can set $N=N_{th}$ and $g=g_{th}$, except when we need the difference $N-N_{th}$ or $g-g_{th}$.

Below and above threshold

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- The stimulated emission rate can be described by a **stimulated carrier lifetime**.

Then, the total carrier recombination rate is

$$R_{tot} = \frac{N}{\tau_{sp}} + \frac{N}{\tau_{nr}} + \frac{N}{\tau_{st}}$$

where

$$\frac{1}{\tau_{sp}} + \frac{1}{\tau_{nr}} = \frac{R_{sp} + R_{nr}}{N}$$

$$\tau_{st} = \frac{N}{v_g g N_p} \approx \frac{N_{th}}{v_g g_{th}} \frac{1}{N_p}$$

- The inverse dependence of the stimulated carrier lifetime on photon density sets up a negative feedback loop, which prevents N from increasing beyond its threshold value.

Steady-state multimode solutions

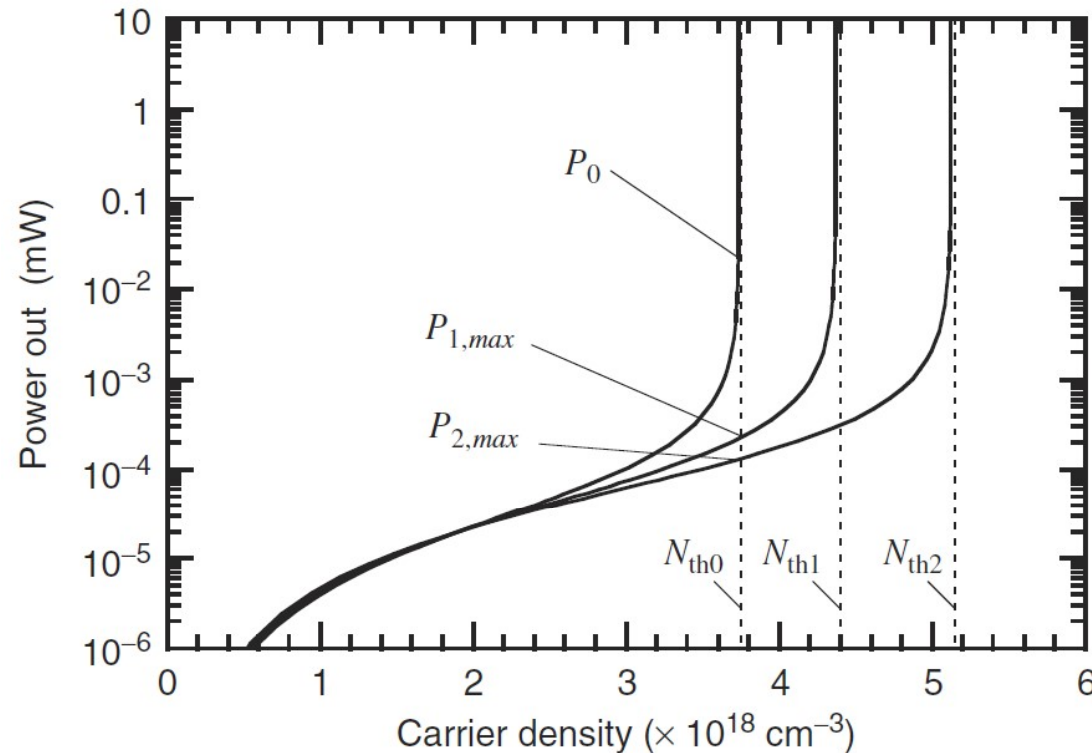
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- In multimode situation, the steady-state photon density for each mode remains the same, while the current must include contributions from all m modes:

$$N_{pm} = \frac{\Gamma_m R'_{spm}}{1 / \tau_{pm} - \Gamma_m v_{gm} g_m}$$
$$I = \frac{qV}{\eta_i} \left(R_{sp} + R_{nr} + \sum_m v_{gm} g_m N_{pm} \right)$$

Steady-state multimode solutions

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- ❑ The power of **side modes** 1 and 2 saturates/clamps when the main mode begins to lasing. The **mode suppression ratio** increases linearly with the power in the main mode.